Downstream Vorticity Measurements from Ultrasonic Pulses

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Abstract

The vorticity distribution behind a profile is obtained from the measurement of the running-time shift, which an ultrasonic pulse experiences in wind tunnel flow. The spatial gradient of the running-time shift is related to the vorticity vector. The method is limited by the assumptions of incompressible flow and a high Reynolds number.

Introduction

The investigation of flow patterns by means of ultrasonic pulses and some initial applications were published in 1982.[1] Running-time measurements were part of an investigation concerning various blade tip shapes of rotor blades.[2] The velocity field induced by the downstream vorticity determines the typical shape of the running-time signals. The mathematical model applied to describe the three-dimensional flow around a profile of finite thickness is a higher-order panel method.[3] The physical assumptions for the fluid are infinitely high Reynolds number and incompressible flow, which lead to a simplified vorticity transport equation.[4]

The method covers the wide range from fixed wings to rotating systems; it is not bound by the need for the mathematical modeling of a given configuration. The prediction of lift is based theoretically on a relation between the running-time shift of an ultrasonic pulse passing the wake and the vorticity contained in it. Mathematical models of selected flowfields allow the theoretical simulation of wind tunnel tests and give some insight into the applicability, accurace and limitations of the method.[5] Preferred applications are comparative measurements of tip shapes or the investigation of rotor wakes.[6] Compared with point-to-point measurements in a flowfield, e.g., by laser anemometry, the method provides integral values in thin and sensitive regions (such as vorticity in the wake). A previous paper presented by the author extends the method of ultrasonic pulse measurements to measurements of vorticity.[7]

Running-Time Shift

The running time is mathematically a well-defined function in the flowfield, and its gradient may be related to the vorticity density along the path of the signal. An ultrasonic pulse passing the flow around a wing experiences a typical shift of its running time. For any point $\boldsymbol{r}$ in the flowfield and any time t the relative velocity $V_{\rm rel}(\boldsymbol{r},t)$ with respect to the wing may be described by the kinematic motion with respect to a frame at rest and the perturbation velocity induced by the moving obstacle. A pulse transmitted at a point $\boldsymbol{r}_T$ into a direction $\boldsymbol{n}(\boldsymbol{r}_T,t)$ is initially propagated at a velocity

$$V_{\rm rel}\boldsymbol{n}(\boldsymbol{r}_T,t) + c_S\boldsymbol{n}(\boldsymbol{r}_T,t) \tag{1}$$

where c_S is the local velocity of sound. Assuming a constant velocity of sound throughout the fluid is equivalent to an almost contant temperature in a wind tunnel, the drift of temperature during a long operating time of the tunnel has to be considered as a time-dependent correction of c_S.

The signal is convected by the fluid; its position after a given time interval results from an integration of the velocity in Eq. (1) and, therefore, is unknown in advance. However, as long as the relative velocity is small compared to the velocity of sound, the final position may be estimated quite accurately. In addition, the pulse widens during its passage through the fluid and finally covers an area large enough to be detected. We approximate the path of the signal by a straight line and prescribe the point $\boldsymbol{r}_R$, where the signal is supposed to be received by a microphone. The given spatial distance D and direction $\boldsymbol{n}$

$$D = |\boldsymbol{r}_R - \boldsymbol{r}_T|, \; \boldsymbol{n} = 1/D(\boldsymbol{r}_R - \boldsymbol{r}_T) \tag{2}$$

between transmitter and receiver defines the running time t_D by integration

$$\frac{\mathrm{d}s}{\mathrm{d}t} = V_{\rm rel}(\boldsymbol{r}(s),t_o) + c_S\boldsymbol{n} \tag{3}$$

along $\boldsymbol{r}(s) = \boldsymbol{r}_T + s\boldsymbol{n}$, which leads to

$$t_D = \int_0^D \frac{\mathrm{d}s}{c_S \pm V_{\rm rel}(s)}, \; V_{\rm rel}\boldsymbol{n} = V_{\rm rel}(s) \tag{4}$$

The arc length s parameterizes the path of the signal from the transmitter point $\boldsymbol{r}_T = \boldsymbol{r}(0)$ to the receiver point $\boldsymbol{r}_R = \boldsymbol{r}(D)$. The constant time $t = t_o$ in Eq. (3) indicates that the fluid is assumed to be frozen. The assumption is justified as long as changes in the flow patter are small during the interval t_D, in which the signal travels through the fluid. With D/c_S as the running-time shift Δt is defined by

$$\Delta t := t_D - D/c_S \tag{5}$$

Δt is exactly the physical variable, which originates from experiments with ultrasonic pulses, as well as from the corresponding theoretical predictions, and reflects basic phys-

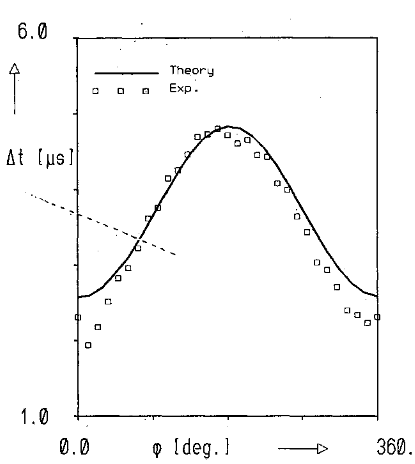

Fig. 1 Running-time shift in unsteady flow.

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ical properties of the flowfield. The running-time shift is of the order of some microseconds and is about 10^{-3} smaller than the running time itself.

Extended measurements of the unsteady running-time shift were performed by W.J. Wagner[2] at a NACA 0012 model. A typical result is shown in Fig. 1. The variation of Δt during one cycle of a pitching motion at a fixed position is plotted for a reduced frequency of 0.75.[5] The position of the ultrasonic beam is half a chord length inside the wake and three cord lengths downstream. The amplitude is 1 deg, and the steady angle of incidence is 5 deg.

Vorticity Measurements

Forming the gradient of t_D in Eq. (4) with respect to the argument $\boldsymbol{r}$ leads to

$$\operatorname{grad} t_D = \int_0^D \operatorname{grad}\left\{\frac{1}{V_{\text{rel}}(\boldsymbol{r}(s))\boldsymbol{n} + c_S}\right\}\mathrm{d}s$$

$$= -\frac{1}{c_S^2}\int_0^D \left\{1 + 2\frac{V_{\text{rel}}}{c_S} + \frac{V_{\text{rel}}^2}{c_S^2}\right\}^{-1}$$

$$\cdot\left[(\boldsymbol{n}\operatorname{grad})V_{\text{rel}} + \boldsymbol{n}\times\operatorname{curl} V_{\text{rel}}\right]\mathrm{d}s \qquad (6)$$

Measurements in air at rest differ from those in a wind tunnel merely by a uniform onset velocity u_o. Therefore, the vectorial derivatives of V_{rel} are allowed to be replaced by the derivatives of the induced velocity V. Assuming small relative velocities compared to the velocity of sound $|V_{\text{rel}}|/c_S \ll 1$, Eq. (6) reads in uniform flow

$$\operatorname{grad} t_D \simeq \frac{-1}{c_S^2}\int_0^D \left[(\boldsymbol{n}\operatorname{grad})V + \boldsymbol{n}\times \boldsymbol{j}\right]\mathrm{d}s \qquad (7)$$

The y-component in Eq. (7) determines the spanwise lift distribution in a blade fixed-coordinate system (x,y,z) where x points downstream, y in spanwise direction and z is oriented perpendicularly to the chord. It should be noted that $\boldsymbol{j}$ is the spatial distribution of vorticity and possesses the unit s^{-1}. It is not to be confused with the vorticity of an infinitely thin sheet that has the unit m/s. We know that the spatial vorticity is spread over a more or less small boundary layer. To relate the spatially distributed vorticity to the theoretically assumed infinitely thin vortex sheet we define as the total vorticity per square unit:

$$j_x^F(x,y,t) := \int_0^D j_x(r',t)\,\mathrm{d}z' \qquad (8)$$

Resolving Eq. (7) for j_x^F we obtain

$$-\frac{1}{c_S^2}\, j_x^F \simeq \frac{t_D(x,y+\Delta y,t) - t_D(x,y,t)}{\Delta y}$$

$$+\frac{1}{c_S^2}\left[V_y(r_R,t) - V_y(r_T,t)\right] \qquad (9)$$

Δy is the distance between two subsequent points of the measurement. The relation between vorticity and running time still implies that the velocity field V_y is known at the respective locations of the transmitter and the receiver. However, test computations[7] show very clearly that a distance of about six chord lengths between emitter and receiver is sufficient to obtain satisfactory results for the vorticity distribution, though the induced velocity V_y is not taken into account.

Finally, lift is obtained from the vorticity measurement by integrating the spanwise distribution $j_x^F(y)$. As an example, the

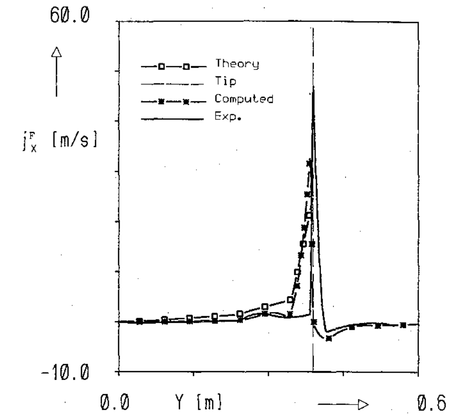

Fig. 2 Spanwise vorticity distribution.

solid line in Fig. 2 shows the spanwise vorticity distribution resulting from the interpolated experimental running-time data of a NACA 0010 profile in steady flow.[7] The broken line filled in with squares shows the vorticity distribution along the trailing edge according to the solution of the integral equation. The broken line with asterisks is the theoretical vorticity distribution computed from the induced flowfield without taking the V_y term into account. The more general case of harmonic oscillations has to be treated according to Eq. (7). The concentration of vorticity in the tip region is obvious and proves the method to be applicable for the investigation of wakes.

Conclusion

The application of the ultrasonic method permits the investigation of wake structures. Currently, experimental experience ranges from steady and unsteady measurements in homogeneous flow to measurements in the wake of a steadily rotating wind turbine. The method has proved to give reliable results for the location as well as for the strength of the trailing vortices. Besides the limitations by basic physical assumptions, the application to rotating systems requires an experimental tool, which allows in addition to the pulse measurement the instantaneous recording of the distance between transmitter and receiver. This magnitude turns out to be a sensitive parameter with respect to the evaluation of the running-time shaft.

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